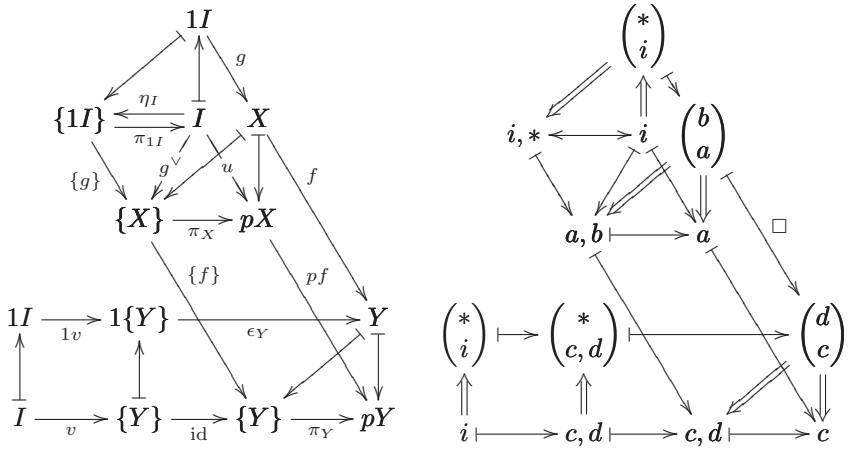
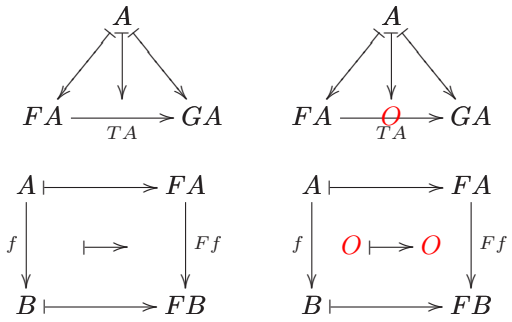
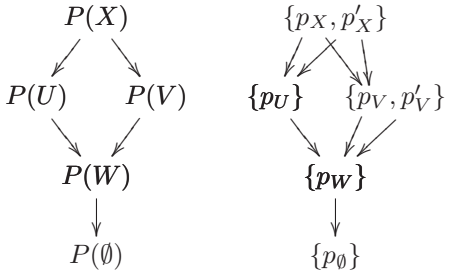
A construction from Bart Jacobs' "Categorical Logic and Type Theory" book (sec. 10.4.7, pp.616–617):



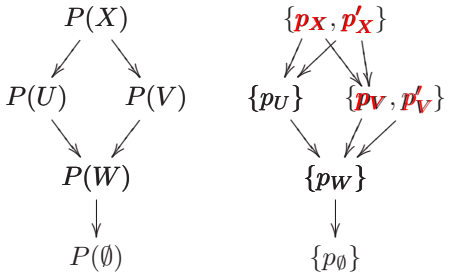
Two examples of diagrams with phantom nodes:



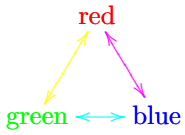
A diagram with relative phantom nodes:



Same, but with the relative phantom nodes in red:



Coolored nodes and arrows:



Some β-reductions from Bierman and de Paiva's
"On an intuitionistic modal logic" paper:

$$\dfrac{N : A \quad \dfrac{\begin{array}{c}[x : A]^1 \quad \Gamma \\ \vdots \\ M : B\end{array}}{(\lambda x : A.M) : A \to B}\supset\mathcal{I};1}{(\lambda x : A.M)N : B}\supset\mathcal{E} \qquad \begin{array}{c}N : A \quad \Gamma \\ \vdots \\ M[x := N] : B\end{array}$$

$$\dfrac{\dfrac{\dfrac{M : A \quad N : B}{\langle M, N\rangle : A \times B}\wedge\mathcal{I}}{\mathsf{fst}\langle M, N\rangle : A \times B}\wedge\mathcal{E}_1 \quad \rightsquigarrow_\beta \quad M : A$$

$$\dfrac{\dfrac{M : A}{\mathsf{inl}(M) : A + B}\vee\mathcal{I} \qquad \begin{array}{ccc}[x : A]^1 & \Gamma & [y : B]^1 \quad \Delta \\ \vdots & & \vdots \\ N : C & & P : C\end{array}}{\mathsf{case}\ \mathsf{inl}(M)\ \mathsf{of}\ \mathsf{inl}(x) \to N \mid\mid \mathsf{inr}(y) \to P}\vee\mathcal{E} \quad \rightsquigarrow_\beta \quad \begin{array}{c}M : A \quad \Gamma \\ \vdots \\ N[x := M] : C\end{array}$$

$$\dfrac{\dfrac{\begin{array}{cc}\Gamma & [\vec{x} : \Box\vec{A}]^1 \\ \vdots & \vdots \\ \vec{M} : \Box A & N : B\end{array}}{\mathsf{box}\ N\ \mathsf{with}\ \vec{M}\ \mathsf{for}\ \vec{x} : \Box B}\Box\mathcal{I};1}{\mathsf{unbox}(\mathsf{box}\ N\ \mathsf{with}\ \vec{M}\ \mathsf{for}\ \vec{x} : \Box B)}\Box\mathcal{E} \quad \rightsquigarrow_\beta \quad \begin{array}{c}\Gamma \\ \vdots \\ \vec{M} : \Box\vec{A} \\ \vdots \\ N[\vec{x} := \vec{M}] : B\end{array}$$

$$\dfrac{\begin{array}{ccc}\Gamma & \Delta & [\vec{x} : \Box\vec{A}\ y : B]^1 \\ \vdots & \vdots & \vdots \\ \vec{M} : \Box A & \dfrac{N : B}{\Diamond N : \Diamond B}\Diamond\mathcal{I} & P : \Diamond C\end{array}}{P[\vec{x} := \vec{M}, \Diamond y := \Diamond N] : \Diamond C}\Diamond\mathcal{E};1 \quad \rightsquigarrow_\beta \quad \begin{array}{cc}\Gamma & \Delta \\ \vdots & \vdots \\ \vec{M} : \Box A & N : B \\ \vdots & \\ P[\vec{x} := \vec{M}, y := N] : \Diamond C & \end{array}$$

"$P[\vec{x} := \vec{M}, \Diamond y := \Diamond N]$" is a shorthand for
"$\mathsf{let}\ \Diamond y \Leftarrow \Diamond N\ \mathsf{in}\ P\ \mathsf{with}\ \vec{M}\ \mathsf{for}\ \vec{x}$".